MATHEMATICS AND SCIENCE SERVICES – UGANDA (MASS)

"MATHEMATICS AND SCIENCE ARE MISCIBLE"

A' Level Mathematics Seminar Questions
Held At Alliance High School Nansana
on Saturday 28th September 2019

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PART A: PURE MATHEMATICS (P425/1)

ALGEBRA

1. (a) (i) Given that $a^3+b^3 = 6ab(a+b)$, prove that $\ln\left(\frac{a+b}{3}\right) = \frac{1}{2}(\ln a+\ln b)$

   (ii) Find x in the equation $5^{\log_{25} x} = 3^{\log_{27} 2x}$

   (iii) Show that $\log(x+y) = \log x + \frac{1}{3}\log\left(1+\frac{3y}{x}+\frac{3y^2}{x^2}+\frac{y^3}{x^3}\right)$

   (iv) Find the root of the equation $2 + \log\sqrt{1+x} + 3\log\sqrt{1-x} = \log\sqrt{1-x^2}$

   (b) (i) Given that the roots of the equation $px^2 + qx + q = 0$ are

   $(\propto_1 - p)$ and $(\propto_2 - q)$. If $(\propto_1 - p) - (\propto_2 - q) = 1$

   Show that $\frac{\propto_1}{\propto_2} = \frac{2p^2+(p-q)}{2pq-(q+p)}$

   (ii) Use raw echelon reduction to solve the following equations simultaneously:

   $2x = 5 + y - z$

   $-3y = 2 - x - 2z$

   $4z = -3 - 2x - y$

   (c) (i) Given that $f(x) = (x-\propto)^2 g(x)$, show that $f^1(x)$ is divisible by $(x - \propto)$

   (ii) A polynomial $p(x) = x^3 + 4ax^2 + bx + 3$ is divisible by $(x-1)^2$. Use the result in (c) (i) above to find the values of **a** and **b**, hence solve the equation $p(x) = 0$

2. (a) Prove by induction $4^{n+3} - 3n - 10$ is divisible by 3 for all positive integral

values of n.

(b) **NASACA** opened up a bank account with shs. 50,000, she deposits the same amount every year and makes no withdrawals. After how many years will she accumulate more than one million shillings on her account if the bank offers 5% compound interest per annum?

3. (a) **MASSAPPE** is a common word used by Ugandans today

   (i) How many possible arrangements of the letters in the word **MASSAPPE** without restriction.

   (ii) How many possible arrangements of the letters in which the two **A's** are together.

   (iii) How many possible arrangements of the letters in which the two **A's** are separated.

   (b) A teacher in **MASS** is to form a team of competitors in mathematics. How many teams of 6 competitors can be formed from a group of 7 boys and 5 girls , if :

   (i) Each team should have atleast 3 boys and a girl.

   (ii) Each team contains at most 3 girls.

4. (a) Find the coefficient of x^3y^4 in the expansion of $(2x - 3y)^7$

   (b) Prove that if x is so small that its cube and higher powers can be neglected, $\sqrt{\frac{1+x}{1-x}} \simeq 1 + x + \frac{x^2}{2}$, by taking $x = \frac{1}{9}$, show that $\sqrt{5} \approx \frac{181}{81}$

5. (a) If x is real and $y = \frac{5x^2+8x+4}{x^2+x}$, show that the curve y cannot lie between - 4 and +4

(b) Solve the following inequalities

(i) $\left|\frac{x^2-4}{x}\right| \leq 3$

(ii) $|x+3a| > 2|x-2a|$

(iii) $\frac{x+2}{x-3} < \frac{x+5}{x-5}$

(c) Sketch the curve $y = \frac{3x+3}{x(3-x)}$ by clearly finding the turning points and asymptotes

6. (a) (i) Show that $\frac{16!}{9!7!} + \frac{2\times16!}{11!6!} + \frac{16!}{11!5!} = \frac{18!}{11!7!}$

(ii) The ratio of the twenty third term of an A.P to the third term exceeds the ratio of the twenty second term to the fourth by 0.5. Given that the sum of the first 25terms is 225, find the first term and common difference of the two progressions which satisfy these conditions.

(b) (i) The first term of a G.P is 3 and the ratio of the third term to the seventh term is 3:4 find the ninth term

(ii) Given a geometric series $\sin 2x + -\sin 2x\cos 2x + \sin 2x\cos^2 2x + \ldots\ldots\ldots\ldots\ldots\ldots$

Find the common ratio and prove that the sum to infinity is tanx

8. (a) (i) Use De- moivre's theorem to show that:

$15\sin^5\theta = \sin 5\theta - 5\sin 3\theta + 10\sin\theta$

(ii) Prove that $3i + 2$ is a root to the equation. $Z^4 - 5Z^3 + 18Z^2 - 17z + 13 = 0$, and hence find all other roots of this equation.

(b)(i) Given that $Z_1 = 6\left(\cos\frac{5}{12} + i\sin\frac{5}{12}\pi\right)$, find Z_1Z_2 and $\frac{Z_1}{Z_2}$ in the form $x + yi$

(ii) Calculate the principle argument of $\frac{(1+i\sqrt{3})^5}{(1-i)^3}$

(iii) Express $Z = \frac{7+4i}{3-2i}$ in the form $p + qi$ where p and q are real.

9. (a) If $Z_1 = 2 + 5i$, $Z_2 = 1 - 3i$ and $Z_3 = 4 - i$. Determine in both Cartesian and polar forms the value o f $\frac{Z_1+Z_2}{Z_1+Z_2} + Z_3$ **correct to 3dps**

(b)(i) Describe the locus given by $|Z+2i| = |2Zi-1|$

(ii) Evaluate $(1+i)^8$

(iii) If $Z = x + iy$ and $|Z-4| \leq 3$, determine the least and greatest value of Z

(c) Given that $z = z + iy$ and $\arg\left(\frac{z}{z-6}\right) = \frac{\pi}{2}$. Show that the locus of Z is $x^2 + y^2 - 6x = 0$

ANALYSIS

1. (a) Find the following integrals

(i) $\int(\ln x)^2dx$

(ii) $\int\frac{2dx}{\sqrt{1-x^2}\cos^{-1}(x)}$

(iii) $\int e^{(e^x+x)}dx$

(iv) $\int 2^{2^{2^x}}.2^{2^x}.2^x dx$

(v) $\int \frac{d\theta}{\sqrt{1-\sin\theta}}$

(vi) $\int x\tan^2 x dx$

(vii) $\int \frac{2\cos x+9\sin x}{3\cos x+\sin x}dx$

(b) Evaluate the following:

(i) $\int_0^{\frac{\pi}{2}} \frac{\cos^4 x}{1+\sin x}$

(ii) $\int_0^{\frac{\pi}{12}} \tan^3 3x dx$

(iii) $\int_0^{\frac{\pi}{2}} \sin 7x \cos 5x dx$

(iv) $\int_0^1 \frac{3-x}{(x+1)(x^2+1)}$

(v) $\int_0^1 \tan^{-1}(2x)dx$

(c) Show that:

(i) $\int x\sin^{-1}(x)dx = \frac{1}{4}(2x^2-1)\sin^{-1}(x) + \frac{1}{4}x\sqrt{(1-x^2)} + c$

(ii) $\int_{\sqrt{3}}^{\infty} \frac{dx}{x\sqrt{1+x^2}} = -\frac{1}{2}\ln 3$

(iii) $\int \frac{x}{2x^2-x+1}dx = \frac{1}{4}\ln\left(2x^2-x+1\right) + \frac{1}{6}\tan^{-1}\left(\frac{4x-1}{3}\right) + c$

(iv) $\int_1^4 \frac{\log_e x}{\sqrt{x}}dx = 8\log_e 2-4$

2. (a) Form and state the order of the formed de given the equations below

(i) $\frac{y}{Ax^2+Bx} = 1$

(ii) $x = \frac{1}{\cot\ (Ay)}$

(b) Solve the following differential equations

(i) $\frac{dy}{dx} = \cos(x\text{-}y)$ given that, $y(\pi) = 0$

(ii) $\frac{dy}{dx} = e^{10t+12y}$ when $y = 0$, $x = 1$

(iii) $\frac{dy}{dx} + 2x\tan x = \sin x,\ y\left(\frac{\pi}{4}\right) = 0$

(iv) $x^2\frac{dy}{dx} = x^2 + y^2 + xy$ given that $y= 0$ when $x = \frac{\pi}{4}$

(v) $\frac{dy}{dx} = 4x - 3y + 2xy - 6$

(c) The population of criminals in Nansana grows at a rate given by the equation $\frac{1}{x}\frac{dx}{dt} = (b - ax)$ given that originally there was one criminal in Nansana. Show that; $\left(\frac{x}{b\text{-}ax}\right)^{1/b} = \left(\frac{1}{b\text{-}a}\right)^{1/b} e^t$

3. (a) Milk tea poured in metallic cup loses heat due to a steady breeze at a rate which is proportional to its temperature θ and also gains heat from a hot fire source directed to it at a rate proportional to time, t,

(i) Write down the differential equations for the temperature θ

(ii) Show that at any time t, $\theta = At + B + Ce^{-kt}$

(b) Find the mean value of $y = \frac{\tan^{-1}(x)}{1-x^2}$ for $0 \le x \le \frac{\pi}{4}$

(c) Determine the volume of the solid generated when the area of the segment cut off by $y = 6$ from the curve $y = x^2 + 2$ is rotated about $y = 6$

4. (a) Differentiate the following from 1st principles

(i) $y = e^{kt}$

(ii) $y = x\ln x$

(iii) $x^4 + \sin x^2$

(iv) $\sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$

(b)(i) Given that $y = (\sin^{-1} x)^2$, show that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 2 = 0$

(ii) If $y = 1 - \cos\theta$ and $x = \sin\theta$, prove that $\left(\frac{d^2y}{dx^2}\right)^2 = \left[1+\left(\frac{dy}{dx}\right)^2\right]^2$

5. (a) Differentiate the following w.r.x

(i) $\frac{3^x+6}{9^x\log_e 3}$

(ii) $\frac{e^{\ln\cos x}\sin x}{\sqrt{\cot x}}$

(b) Given that $x = \sec\theta + \tan\theta$, $y = \operatorname{cosec}\theta + \cot\theta$. Show that $x + \frac{1}{x} = 2\sec\theta$ and $y + \frac{1}{y} = 2\operatorname{cosec}\theta$. Hence show that $\frac{dy}{dx} = -\left(\frac{1-y^2}{1+x^2}\right)$

(c) If y= $(\sec x+\tan x)^2$, show that $\cos x\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = 2\tan x$

6. (a) A water tank of a uniform cross sectional area $2cm^2$ has a tap at the back. When the tap is opened water flows out at a rate proportional to the depth of water in the tank.

   (i) Show that $\frac{dh}{dt} = -\lambda h$

   (ii) If the depth of water is 1cm when the tap is opened, find the time it will take until the depth is 50cm, assume $\lambda = \frac{1}{50}$

   (b) The curve has the equation $x - y = (x + y)^2$. It is also given that the curve has only one turning point.

   (i) Show that $1 + \frac{dy}{dx} = \frac{2}{2x+2y+1}$

   (ii) Hence or other show that $\frac{d^2y}{dx^2} = \left(1+\frac{dy}{dx}\right)^3$

   (iii) Deduce whether this turning point is maximum or minimum

   (c) Find an approximate value for $\sqrt[3]{64.96}$

   (d) Use Maclaurins theorem to expand $(1 - 3x + 5x^2)$ up to the third non – zero term

TRIGONOMETRY

1. (a) Solve:

   (i) $4\sin^2\theta + 8\cot^2\theta - 5\operatorname{cosec}^2\theta = 0$ for $0 \le x \le 360^0$

   (ii) $\cot^2\theta - 2\cot\theta\operatorname{cosec}\theta = 0, 0 \le x \le 360^0$

(iii) $2^{\tan^2 x+8} - 32(2^{\tan x}) + 1 = 0$ where $0 \leq x \leq 180^0$

(b) Prove that :

(i) $\left(\frac{1+\sin 2x}{1-\sin 2x}\right)^{1/2} = \frac{1+\tan x}{1-\tan x}$

(ii) In any triangle ABC, $\tan\frac{A}{2}\tan\frac{B}{2} + \tan\frac{B}{2}\tan\frac{C}{2} + \tan\frac{C}{2}\tan\frac{A}{2} = 1$

(iii) $\frac{\sin 2x - 1 - \cos 2x}{2(1-\sin 2x)} = \frac{1}{\tan x - 1}$

2. (a) (i) Show that $\sin^{-1}\left(\frac{3}{5}\right) - 2\tan^{-1}\left(\frac{1}{5}\right) = \sec^{-1}\left(\frac{65}{63}\right)$

(ii) Prove that $\tan^{-1}(x) + \cot^{-1}(x) = \frac{\pi}{2}$

(iii) Show that $\sin\left[2\sin^{-1}(x) + \cos^{-1}(x)\right] = \sqrt{(1-x^2)}$

(b) Express $2\sqrt{3}\sin\theta\cos\theta + 2\cos\theta^2$ in the form $a\sin(2\theta+a) + b$, hence solve the equation $2\sqrt{3}\sin\theta\cos\theta + 2\cos^2\theta = 3$ for $0^0 \leq \theta \leq 360^0$

(c) Find a positive value of θ that satisifies the equation $\tan^{-1} 3\theta + \tan^{-1}\theta = \frac{\pi}{4}$

VECTORS

1. (a) Find the Cartesian equation of the plane

$$r = (1+3\lambda+3\mu)i + (1+\lambda+4\mu)j + (\mu+\lambda)k$$

(b) (i) Find the equation of a line passing through points A (1, 2, 5) B(2,1,0) and C(5,3,2)

(ii) Determine the perpendicular distance from the points B(1,13) to the line:

$$\frac{x+4}{2} = \frac{y+1}{3} = \frac{z-1}{3}$$

(c) (i) Find the perpendicular distance between the planes $6x - 3y + 2Z + 4 = 0$ and $6x - 3y + 2z - 12 = 0$

(ii) Find the acute angle between the planes in (c) (i) above.

2. (a) Find the equation of the plane passing through the origin and parallel to the lines

$$\frac{x+2}{3} = \frac{y-1}{4} = \frac{z+1}{5} \text{ and } \frac{x-3}{4} = \frac{y-2}{-5} = \frac{z+1}{1}$$

(b) Find the possible values of **t** given that the vectors $ti + 4j + (2t+1)k$ and $(t+2)i + (1-t)j - k$ are perpendicular to each other.

(c) Show that the line $\frac{x-2}{2} = \frac{y-2}{4} = \frac{z-3}{3}$ and plane $r\begin{pmatrix}4\\-1\\3\end{pmatrix} = 4$ are parallel and fin the perpendicular distance of the line from the plane.

(d) The position vectors of vertices of triangle are O, r and S where O is the origin , show that its area (A) is given by $A = \frac{\sqrt{|r|^2|s|^2-(r.s)^2}}{2}$, Hence find the area of a triangle where $r = \begin{pmatrix}2\\3\end{pmatrix}$ and $s = \begin{pmatrix}1\\-4\end{pmatrix}$

GEOMETRY

1. (a) Show that the Cartesian equation whose polar equation is given by

$r^2 = a(\sec 2\theta + 1)$ is $x^4 - y^4 = 2ax^2$

(b) Find the angle between the lines:

$ax - by + c = 0$ and $(a-b)x + (a+b)y + d = 0$

(c) Given that the curve $y^2 = x^3$

(i) Obtain the equation of the normal at the point (t^2, t^3)

(ii) Show that the equation of the normal at the point where $t = ½$ is

$32x + 24y - 11 = 0$

(iii) Find the perpendicular distance for the point (-1, 2) to this normal

2. The point $p(ap^2, 2ap)$ lies on a parabola $y^2 = 4ax$ the normal at p cuts the x-axis at Q

(a) Find the coordinates of Q

(b) R divides PQ externally in the ratio 2 : 1 ,

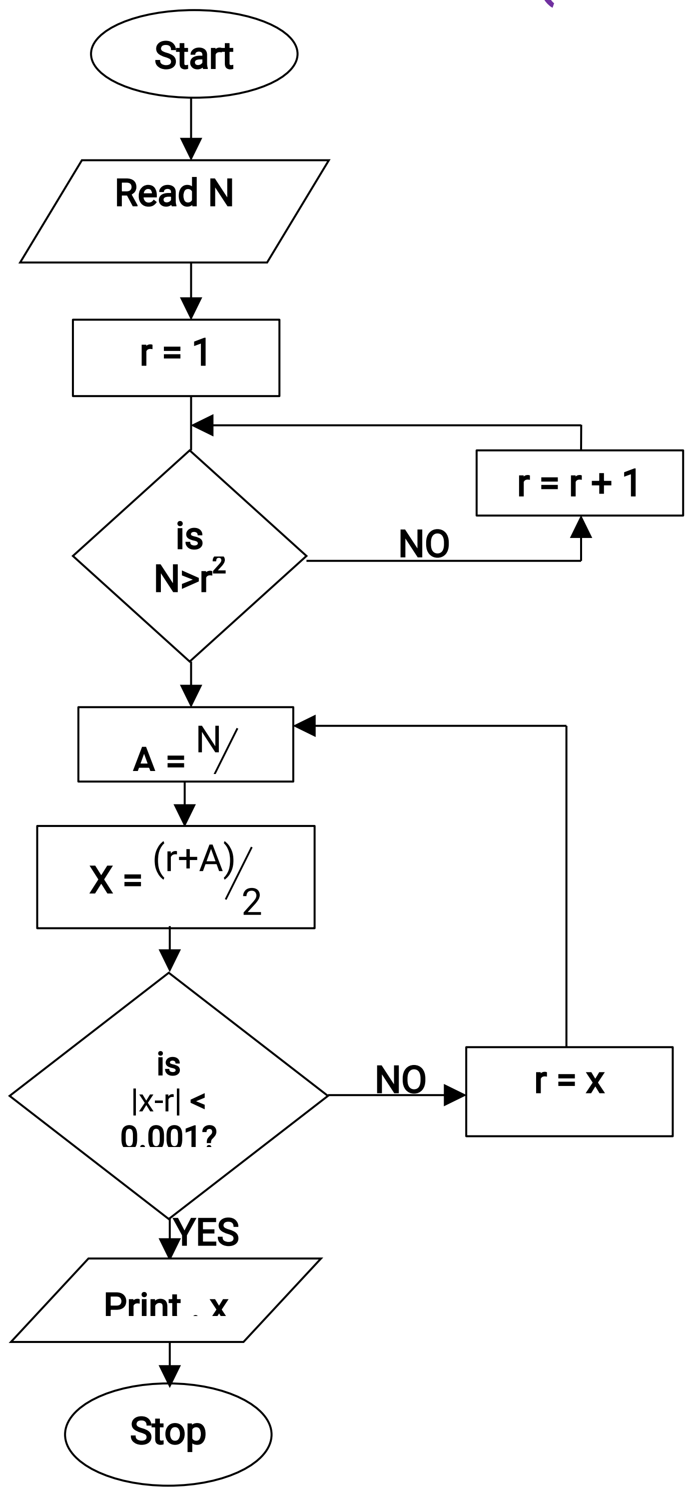
Show that the locus of R is $y^2 + 16a^2 = 4ax$

3. (a) The points $P(ap_1^2, 2ap_1)$ and $Q(ap_2^2, 2ap_2)$ are on a parabola $y^2 = 4ax$. OP is perpendicular to **OQ** where O is the origin , show that $p_1p_2 + 4 = 0$

(b) The normal to the rectangular xy = 8 at a point (4,2) meets the asymptotes at M and N. Find the length **MN**.

PART B: APPLIED MATHEMATICS (P425/2)

1.



(i) Perform a dry run for N = 20 , showing clearly the contents of each store and values of x printed put

(ii) What process does the flow chat represent?

2. (i) Use the trapezium rule to estimate the area of $y = 3^x$ between the x – axis ,

x =1 and x = 2 using 5 strips . **Give your answer to 4 s.f**

(ii) Find the exact value of $\int_1^2 3^x dx$

(iii) Find the percentage error in the calculations (i) and (ii) above

3. (a) In an experiment to measure the rate of cooling of an object, the following temperature (^{0}C) against times, T(s) were recorded.

Temperature	80	70.2	65.8	61.9	54.2
Time, T	0	10	15	20	30

Use linear interpolation / extrapolation to find:

(i) The values of θ when T = 18s

(ii) T when $\theta = 60^0$

(b) In the table below is an extract of part of logx to base 10, $\log_{10} x$

x	80.0	80.20	80.50	80.80
$\log_{10} x$	1.9031	1.9042	1.9059	1.9074

(i) Use linear interpolation / extrapolation to estimate $\log_{10} 80.759$

(ii) The number whose logarithm is 1.90388

4. (a) If z = sinx Determine the expressions for the absolute error and maximum relative error

(b) Given that the error in measuring an angle is 0.5^0. Find the maximum possible error in $\frac{\sin x}{\cos x}$ if $x = 30^0$

5. (a) It is known that an examination paper is marked in such a way that the standard deviation of the marks is 15.1. In certain school 80 candidates take the exam and they have an average mark of 57.7 find,

    (i) 95% and

    (ii) 9% confidence limits for the mean mark in the examination

   (b) The table below shows the distribution of weights of a random sample of the 26 times taken from large consignment.

Weight	97	98	99	100	101	102
Frequency	2	1	2	3	6	2

   Assuming the weights are normally distributed determine the 93% confidence interval for the mean weight of all the tins.

6. A continuous random variable x has the distribution function

$$f(x) = \begin{cases} 3kt\left(1-\frac{x^2}{3}\right) & 0 \le x \le 1 \\ 1 & x > 1 \end{cases}$$

Determine :

(i) the value of k

(ii) the probability density function of x

(iii) the mean of x

7. (a) For the probability that a female and male teachers pass the intervals $^{1}/_{3}$ and $^{2}/_{5}$ respectively. Assuming these events are independent, determine the probability that both pass the intervals.

(b) In a certain university , 75% of the students are full – time students , 45% of the students are female , 40% of the students are male full time students ., find the probability that

(c) A student chosen at random from the students in the university is a part time student

(d) A student chosen at random from all students in the university is female and part time student

(e) A student chosen at random from all the female students in the university is a part time student

8. The table below shows the marks obtained by students of maths in a certain school

Marks	**Number of students**
30 - < 40	02
40 - < 50	15
50 - < 55	10
55 - < 60	11
60 - < 70	30
70 - < 80	29
90 - < 100	3

(a) Calculate e the mean and standard deviation

(b) Draw an O - give for the data

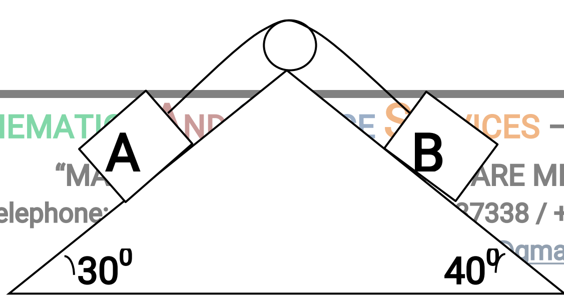
(c) From the graph determine

(i) The median

(ii) The 90th percentage

(iii) Interquartile range

9.



Two particles A and B rest on an inclined plane of a fixed triangular wedge as shown above. A and B are connected by a light inextensible string which passes over a smooth fixed pulley at C. The faces of the wedge are smooth and A and B are both 7kg masses.

Find the force exerted by the string on the pulley at when the system is moving freely with both particles in contact with the wedge.

10. A smooth inclined plane of length **L**, and height **h**, is fixed on a horizontal plane. Show that the velocity with which a particle must be projected down the plane from the top in order that it may reach the horizontal plane in the same time as a particle let fall from the top is

$$u = \frac{L^2 - h^2\left(\sqrt{\frac{g}{2h}}\right)^{1/2}}{L}$$

11. (a) A particle moving with a velocity $(2i+3j)ms^{-1}$ is accelerated uniformly at the rate of $3ti-2j)ms^{-1}$ from the origin

Find the:

(i) Speed reached by the particle at t = 45

(ii) Distance travelled by the particle at t = 25

(b) A particle moving in straight line with uniform acceleration, a, passes a certain point with a velocity , u, three seconds later another particle moving in the same line with constant acceleration $\frac{4}{3}a$, passes the same point with a velocity $\frac{1}{3}$u. The first particle is over taken by the 3 second when their velocities are respectively 8.1 and 9.3ms^{-1}. Find the values of **u** and **a** and also the distance travelled from the point.

Just the beginning…………….

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